Assignment: Module 9 Name: Hoyoung kim

Disclaimer: This is my work, not that of others

Total Score: 55

1. 25
2. 10
3. 10
4. 10

1. The following system of equations is designed to determine the concentrations (c’s in

g/m3

) in a series of coupled, well-mixed tanks as a function of mass input to each

tank. The right-hand side of the equations below represent these inputs in g/day.

15𝑐1 − 3𝑐2 − 𝑐3 = 4000

−3𝑐1 + 18𝑐2 − 6𝑐3 = 1200

−4𝑐1 − 𝑐2 + 12𝑐3 = 2350

a. (5 pt) Determine the inverse of the coefficient matrix. (You can use Python to

solve the inverse.)

import numpy as np

x = np.matrix('15, -3, -1;-3, 18, -6;-4,-1,12')

x = np.linalg.inv(x)

print(x)

[[0.07253886 0.01278066 0.01243523]

[0.02072539 0.06079447 0.03212435]

[0.02590674 0.00932642 0.09015544]]

b. (10 pt) Use the inverse to determine the solution. (Do this by hand.)

[[0.07253886 0.01278066 0.01243523] [4000]

[0.02072539 0.06079447 0.03212435] [1200]

[0.02590674 0.00932642 0.09015544]] [2350]

[334.7150

231.3471

326.683948]

c. (10 pt) Determine how much the rate of mass input to tank 3 must increase to

induce a 10 g/m3

0.01243523 \* x = 10

X = 804.16687

rise in the concentration in tank 1.

2. (10 pt) Determine ‖𝐴‖𝑓, ‖𝐴‖1

, and ‖𝐴‖∞ for

𝐴 =

8 2 −10

−9 1 3

15 −1 6

‖𝐴‖𝑓 = sqrt (EA^2) 521

‖𝐴‖1 = max sum of each column 32

‖𝐴‖∞ = max sum of each row 22

3. (10 pt) Solve the following system using three iterations of the Gauss-Seidel method.

If necessary, rearrange the equations. Show all the steps in your solution. At the end

of your computation, compute the true error of your final results. (Do this by hand.)

7𝑥1 − 𝑥2 = 5 (5 + x2)/7 = x1

3𝑥1 + 8𝑥2 = 11 (11- 3x1)/8 = x2

7 1 = x1 (5 + 0)/7 = x1… x1 = 5/7. (11 - 3(5/7) )/8 = 1.107142871

3 8 = x2 (5 + 1.107142871)/7 = 0.872448981571 (11- 3x1)/8 = 1.04783163191 (5 + 1.107142871)/7 = 0.863975947416. (11- 3x1)/8 = 1.05100901972

Relative error

(0.872448981571 - 0.863975947416) /0.863975947416 = 0.009807…

(1.04783163191- 1.05100901972)/ 1.05100901972 = −0.0030231…

Realistic error

By using substitution, we find that actual value of x1 is 0.8644067

By using substitution, we find that actual value of x2 is 1.0508474

(0.863975947416-0.8644067)/0.8644067 is 0.000498321662708

(1.05100901972-1.0508474)/ 1.0508474 is 0.000153799419402

4. (10 pt) Use the Gauss-Seidel method (a) without relaxation and (b) with relaxation

(𝜆 = 1.2) to solve the following set of linear equation to meet an error tolerance of

𝜀𝑠 = 5%. If necessary, rearrange the equations to achieve convergence.

2𝑥1 − 6𝑥2 − 𝑥3 = −38

−3𝑥1 − 𝑥2 + 7𝑥3 = −34

−8𝑥1 + 𝑥2 − 2𝑥3 = −20

−8𝑥1 + 𝑥2 − 2𝑥3 = −20 x1 = (-20 +2x3-x2)/-8

2𝑥1 − 6𝑥2 − 𝑥3 = −38 x2 = (-38+x3-2x1)/-6

−3𝑥1 − 𝑥2 + 7𝑥3 = −34 x3 = (-34+3x1+x2)/7

x1 = (-20 +2(0)-(0))/-8 = 2.5

x2 = (-38+x3-2x1)/-6

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Iteration | x1 | x2 | x3 |  |  |  |
| 0 | 0 | 0 | 0 |  |  |  |
| 1 | 2.5 | 7.16666667 | -2.7619048 | 100% | 100% | 100% |
| 2 | 4.08630952 | 8.15575397 | -1.9407596 | 39% | 12% | 42% |
| 3 | 4.00465916 | 7.99167966 | -1.9991918 | 2.00% | 2.10% | 2.90% |
| 4 | 3.99875792 | 7.99945128 | -2.0006107 | 0.10% | 0.10% | 0.10% |